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Step response of an imaging system illuminated by partially polarized and partially coherent light

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ABSTRACT

In electronic engineering and control theory, a step response plays a fundamental role to evaluate the response of a system to its inputs change from zero to one in a very short time. From a practical standpoint, knowing how the system responds to a sudden input is important because large and fast deviations from the long term steady state may have extreme effects on the component itself and on other portions of the overall system dependent on this component. In addition, knowing the step response of a dynamical system gives information on the stability of such a system, and on its ability to reach one stationary state when starting from another.

Keywords: Step response, polarization imaging, partially polarized and partially coherent light

1. INTRODUCTION

In physical optics, imaging of an edge object can be regarded as the step response of an optical system and a sharp edge has been used widely in the evaluation of instruments and some photographic systems. From another point of view, polarization imaging is an advanced imaging technology at present, because it can detect the polarization information of the target and has been widely concerned. Many applications have been developed in the fields, such as biomedical diagnostics and target detection and identification. Although various approaches and applications of polarization imaging have been studied in the last decades, few investigations have been made to the step response of a polarization imaging system. In this paper, we study the Stokes images of an edge object under illumination of a light source with arbitrary polarization and coherence. Similar to the parameters used in the conventional step response as time response, we propose several quantities related to the spatial behavior of the imaging system[1~5].Three new parameters, including rise distance, settling distance, overshoot, are adopted to describe the space-dependence of the imaging system response to an edge object under varying illumination conditions of coherence and polarization.

Step response refers to the change of the output of a system when its input is a unit step function. In the field of electronic engineering or automatic control, step response refers to the time-domain characteristics of the output when the input of the system changes from 0 to 1 in a short time. This concept can be extended to optical systems using abstract mathematical concepts, whose properties are represented by evolutionary parameters[6~12].The research of step response in optical system will bring some experience to target detection, biomedicine and other fields。

2. IMAGING SYSTEM AND NEW DEFINITION

In this section, the polarization imaging system with partially coherent light for the edge object is given. Based on the imaging system, three new parameters will be introduced to describe the step response in the optical system. A simple polarization optical imaging system and the schematic diagram of the new definition are shown below:

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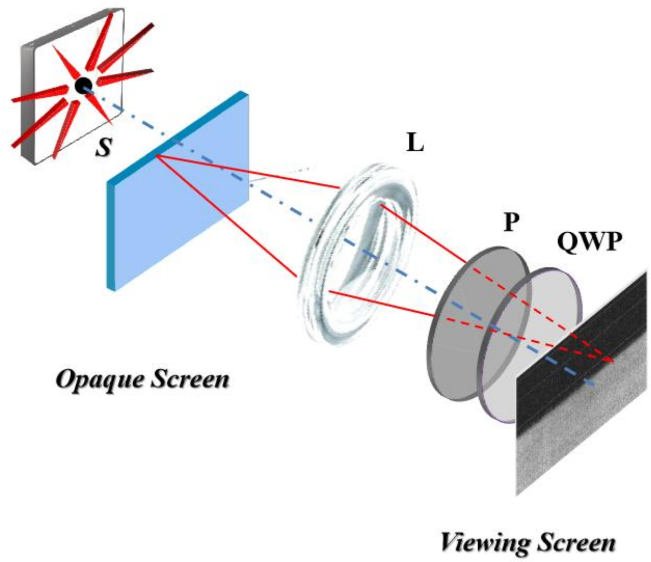

Fig 1.1 Schematic diagram for edge objects illuminated by partially coherent and partially polarized light

Figure 1.1 is a schematic diagram of an edge object for polarization imaging. The object includes a polarization imaging schematic diagram of the edge object illuminated by partially coherent and partially polarized light (S). The Stokes parameters in the image plane (Viewing Screen) are detected by using a polarizer (P) and a quarter wave plate (QWP).

The light source (S) is a square light source with any degree of coherence. The distance between the light source and the edge object is represented by z_o, and the distance between the lens and the image plane is represented by z_i

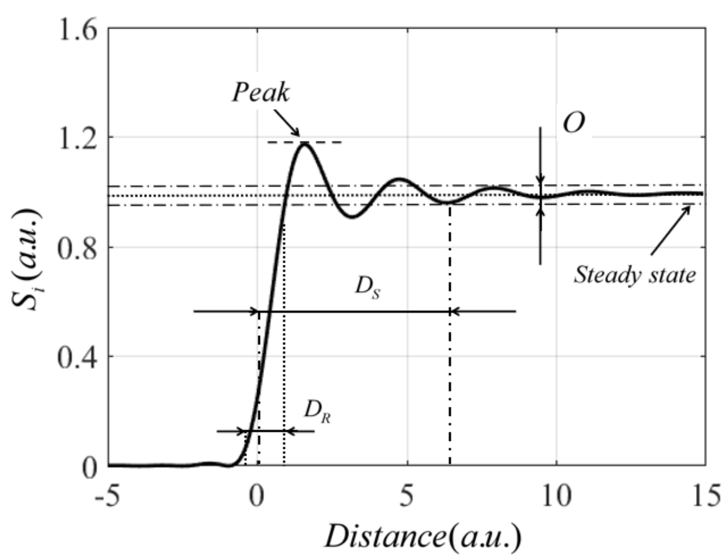

Fig 1.2 Illustration of three new parameter definitions

In this paper, three new parameters are introduced to describe the step response of optical system: Rise distance, setting distance and overshoot. In Fig 1.2, they are expressed by the abbreviations D_S, D_R and O respectively. The relevant parameters are defined as follows:

Peak — Peak absolute value of signal.

Steady state — When the distance tends to infinity, it has a stable output state.

Rise Distance — The distance required for the response to rise from 10% of the steady-state response to 90%.

Settling Distance —The numerical difference between the signal and the steady-state response can always be kept within the required distance of 2%

Overshoot — Percentage overshoot, relative to steady-state response and peak.

Studying the step response of optical system can make a deeper understanding of the characteristics of the system, because when the input optical signal is stable in a long distance and changes rapidly and substantially, the characteristics of each part of the optical system can be obtained by evaluating several parameter values of the optical system, and the stability of the system can also be obtained.

3. POLARIZATION IMAGES OF AN EDGE OBJECT

In recent decades, edge object images illuminated by partially coherent light have been widely studied due to its theoretical and practical value . Although various edge tracking analysis methods have been developed, the research on the step response of polarization imaging system is rare. In this paper, we study the polarization imaging law of edge objects under the illumination of arbitrary polarization and coherent light source. Based on the step response method of evaluated signal in the field of electricity and automatic control, several parameters for evaluating optical system are given as the basic principle for evaluating the performance of polarization imaging.

The image plane optical vibration equation of an optical system in one-dimensional space can always be expressed by the convolution of the object surface equation and the unit impulse response of the system:

$$U^{im}(u) = U^{ob}(u) * h(u) = \int_{-\infty}^{\infty} h(x) U(u-x) dx \tag{1}$$

The light intensity of the object surface can be expressed as:

$$\begin{aligned} I^{im}(u) &= < U^{im}(u)[U^{im}(u)]^* > \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x_1, y_1) h^*(x_2, y_2) U(u-x)[U(u-x)]^* dx_1 \mathrm{d}x_2 \end{aligned} \tag{2}$$

The light vibration equation of the object surface can be expressed as the superposition of the light vibration equation of the light source and the pupil function

$$U^{ob}(x) = P(x) U^S(x) = step(x) U^S(x) \tag{3}$$

By introducing it into formula 2, the light intensity of the image plane can be obtained as follows:

$$I^{im}(u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x_1, y_1) h^*(x_2, y_2) step(u-x_1) step(u-x_2) < U^S(u-x_1)[U^S(u-x_2)]^* > dx_1 dx_2 \tag{4}$$

For a rectangular pupil, its unit impulse response $h(x,y)$ can be expressed by the sinc function, and its transfer function and $h(x,y)$ in a two-dimensional space can be expressed as:

$$H(f_x, f_y) = rect(\frac{\lambda z_i f_x}{2w}) rect \frac{\lambda z_i f_y}{2w}) \tag{5}$$

$$\mathrm{h}(x, y) = (\frac{2w}{\lambda z_i})^2 \sin c(\frac{2wx}{\lambda z_i}) \sin c(\frac{2wy}{\lambda z_i}) \tag{6}$$

where $2w$ is the size/length of a rectangular shape of pupil, λ represents wavelength. In a two-dimensional space, its intensity can be expressed as:

$$\begin{aligned} I^{im}(u) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x_1, y_1) h^*(x_2, y_2) \mathrm{step}(u-x_1) \mathrm{step}(u-x_2) < U^S(u-x_1)[U^S(u-x_2)]^* > dx_1 dx_2 \\ &= (\frac{2w}{\lambda z_i})^4 \int_{-\infty}^{\mathrm{u}}\int_{-\infty}^{u}\int_0^1\int_0^1 \sin c(\frac{2wx_1}{\lambda z_i}) \sin c(\frac{2wx_2}{\lambda z_i}) \sin c(\frac{2wy_1}{\lambda z_i}) \sin c(\frac{2wy_2}{\lambda z_i}) J(x_1 - x_2) dy_1 dy_2 dx_1 dx_2 \end{aligned} \tag{7}$$

Of which

$$\begin{aligned} J(x_1 - x_2) &= < U^S(u-x_1)[U^S(u-x_2)]^* > \\ &= A_{S_i} \exp\left\{-(x_1 - x_2)^2 / (2r_C^2)]\right\} \end{aligned} \tag{8}$$

In the theory of partial coherence and partial polarization, the generalized Stokes parameters defined by [10,11]:

$$\begin{cases} S_0^{in}(x_1,x_2)=\left\langle \tilde{E}_x(x_1,t)\tilde{E}_x^*(x_2,t)+\tilde{E}_y(x_1,t)\tilde{E}_y^*(x_2,t)\right\rangle \\ S_1^{in}(x_1,x_2)=\left\langle \tilde{E}_x(x_1,t)\tilde{E}_x^*(x_2,t)-\tilde{E}_y(x_1,t)\tilde{E}_y^*(x_2,t)\right\rangle \\ S_2^{in}(x_1,x_2)=\left\langle \tilde{E}_x(x_1,t)\tilde{E}_y^*(x_2,t)+\tilde{E}_x^*(x_2,t)\tilde{E}_y(x_1,t)\right\rangle \\ S_3^{in}(x_1,x_2)=\left\langle i[\tilde{E}_x^*(x_2,t)\tilde{E}_y(x_1,t)-\tilde{E}_x(x_1,t)\tilde{E}_y^*(x_2,t)]\right\rangle \end{cases} \tag{9}$$

$S_i^{in}(x_1,x_2),\ i=0,1,2,3$ presents the normalized Stokes parameter in the object plane, where $<\cdots>$ indicates ensemble average, $\tilde{E}_x$ and $\tilde{E}_y$ are electric field vectors in different directions. The normalized Stokes parameter $\hat{s}_i$ will be used to represent the influence of polarization on the imaging system.

From this we can get

$$S_i^{im}(u)=\sqrt{2}(2w/\lambda z_i)^2\hat{s}_i A_C\int_{-\infty}^{u}\int_{-\infty}^{u}\sin c(2wx_1/\lambda z_i)\sin c(2wx_2/\lambda z_i)\mu_i^S(x_1-x_2)dx_1dx_2,\quad (i=0\sim 3) \tag{10}$$

Under the illumination of incident light source with different degree of coherence, we have

$$\mu_i^S(x_1-x_2)=A_{S_i}\ exp\left\{-(x_1-x_2)^2/(2r_C^2)]\right\} \tag{11}$$

Different r_c will give different coherence width, which could influence the value of integral.

After normalization by the $S_i^{im}(\infty)$, we can get the change of rise distance, settling distance, overshoot with change of coherence area.

$$S_i^{im}(u)\propto\hat{s}_i\int_{-\infty}^{u}\int_{-\infty}^{u}\sin c(2wx_1/\lambda z_i)\sin c(2wx_2/\lambda z_i)\mu_i^S(x_1-x_2)dx_1dx_2,\quad (i=0\sim 3) \tag{12}$$

$\mu_i^S(x_1-x_2)$ is integrated and represented by $A_C(r_c)$, where $A_C(r_c)$ is the normalized coherent area.

$$A_C(r_c)=\int_{-\infty}^{\infty}|\mu(\Delta x)|^2 d\Delta x \tag{13}$$

It will be more universal to use the normalized coherent area to express it. The change of the coherent area also means that the coherence degree of the light source has changed. In the formula, it can be concluded that the Stokes intensity in four directions on the image plane is related to the coherent area, pupil type and polarization state. If it is nonpolarized, it can be seen that the intensity distribution is only in the direction of S_0, while it is 0 in the other three directions. The Stokes intensity is positively correlated with the selection of normalized Stokes parameters.

4. DESCRIPTION OF STEP RESPONSE OF OPTICAL SYSTEM WITH NEW PARAMETERS

In this section, the distribution of Stokes cross intensities in four directions in partially polarized state with coherent normalized area and different distances is given and described by three newly defined parameters in this paper. In order to give a more general form, the abscissa and ordinate in the subsequent results are normalized, and their units are expressed in dimensionless units.

The study of step response in optical system can make the change process of "from nothing to existence" clearer, and make the optical imaging process more complete description and evaluation. In electricity, when an excitation signal is a unit step signal, its zero state response is the step response in the circuit. In an optical system, we can use a similar method to describe it. In the step response of the optical system, rise distance, settling distance, overshoot is adopted to represent the optical path of the optical system from scratch to existence, the optical path to reach the stable path, and the relationship between the maximum value and the stable value.

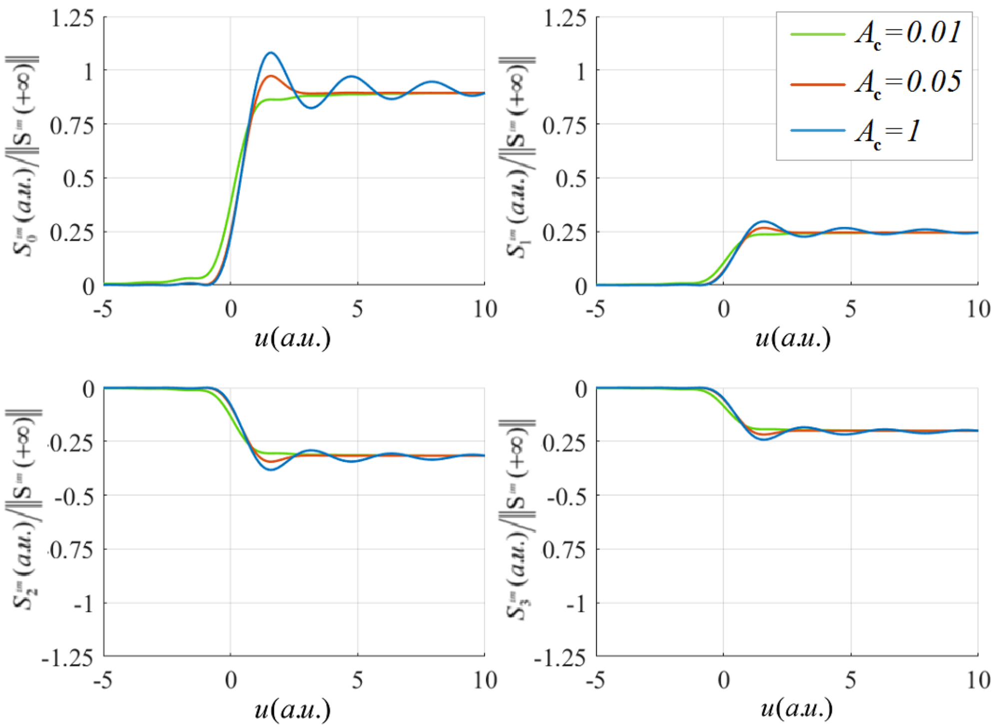

Fig 2 The transformation of Stokes intensity in four directions with normalized coherent area

Fig 2 shows the variation of Stokes light intensity with normalized coherent area in partially polarized state. Fig2 uses a normalized dimension $\mathbf{S}_i^{im}(x)/\|\mathbf{S}^{im}(0)\|$, where $\|\mathbf{S}^{im}(0)\|=\sqrt{\sum_i|\mathbf{S}_i^{im}(0)|^2}$ The four Stokes parameters are 0.89, 0.24, - 0.31 and - 0.2, respectively. It can be seen that when the normalized coherent area is 1, and when this parameter is greater than this value, the curve is expressed as fully coherent state, and its curve jitter amplitude is the largest, and the distance required for stability is reached Longest. In the range of 0.9 to 0.1, it is always close to the state of complete polarization and close to the state when the coherent area is 1. When it is less than 0.1, the curve changes rapidly until it reaches 0.01.

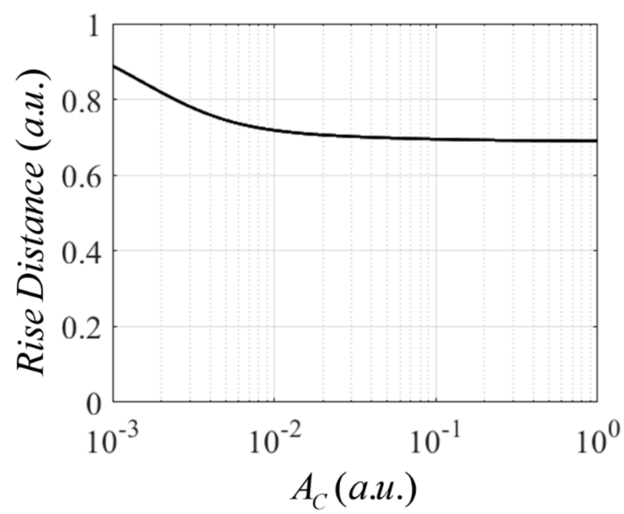

Fig 3.1 Influence of normalized coherent area on Rise Distance

Fig 3.1 shows the relationship between the rise time and the normalized coherent area, because it changes more sharply from 0 to 0.1, while changes in the rest parts are relatively slow. The abscissa adopts logarithmic coordinate. This graph represents the normalized change diagram, so when the abscissa is 0, the corresponding ordinate is 1. It can be seen from the figure that when the light source changes from completely incoherent to coherent, the optical path from 0 to 90% of the stable value in the jump response becomes shorter and shorter, and only when the incoherent part changes violently, it has little effect when it is close to completely coherent light.

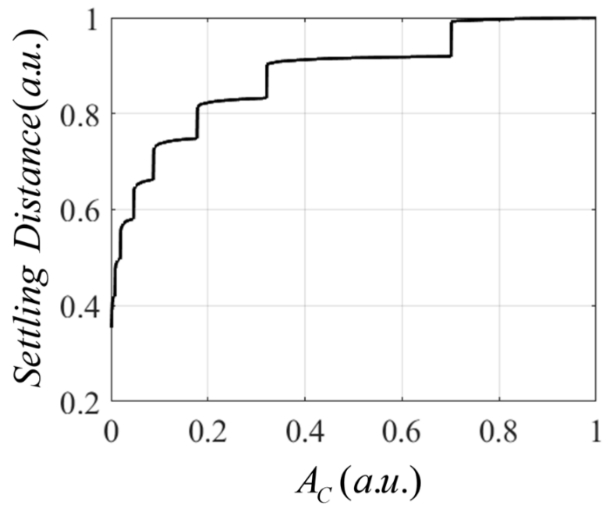

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Fig 3.2 Influence of normalized coherent area on Settling Distance

Fig. 3.2 shows the relationship between setting distance and normalized coherence area. Since it changes more stably in each segment compared with the other two parameters, the abscissa and ordinate are linear coordinates. However, it can still be seen that the smaller the coherence area, the more intense the change is. This is where the three parameters are consistent. The change trend of setting distance presents a "ladder" shape, and the closer to the coherence, the longer the setting distance in the step response.

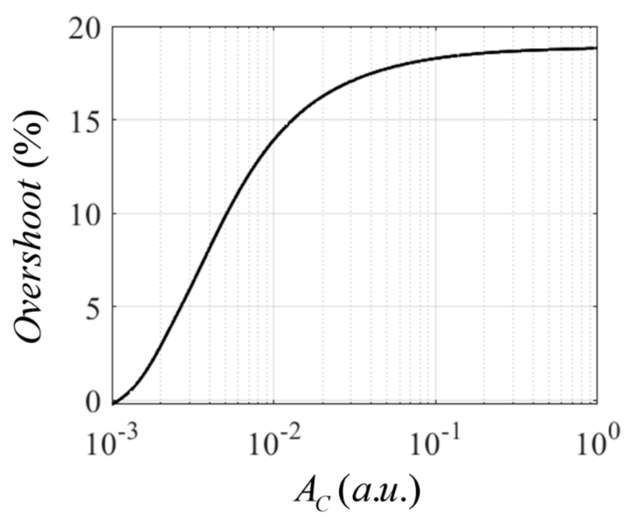

Fig 3.3 Influence of normalized coherent area on Overshoot

Fig. 3.3 is the graph of the relationship between overshoot and normalized coherent area. It also changes dramatically when the coherence area is small, so it is expressed in logarithmic coordinates. Overshoot shows the proportion of the maximum value higher than the stable value in the step response. In addition, the changes of the three newly defined parameters are independent of the polarization state of the light source.

4. CONCLUSION

Three parameters, rise distance, setting distance and overshoot, which are used to describe the step response of an optical system, are proposed, and they all change monotonically with the coherence area; When the normalized coherent area is only about 0 to 0.1, the transformation of Stokes intensity distribution on the image plane is intense, but in the range of 0.1 to 1, the transformation is slow and tends to the Stokes intensity distribution of fully coherent state; Both rise distance and overshoot change sharply from 0 to 0.1, and change slowly after 0.1;The change of polarization does not affect the distribution of the three parameters.

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